

# Phenomenology of neutrino oscillation —Brief overview and its relevance to tau physics—

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## Abstract

Status of neutrino oscillation study is briefly reviewed. Some aspects relevant to tau physics are also described.

**Keywords:** neutrino oscillation, parameter degeneracy, new physics

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## 1. Status of study of the three flavor neutrino oscillation

In the standard framework of three massive neutrinos, the parameters which describe neutrino oscillation phenomena are three mixing angles  $\theta_{12}$ ,  $\theta_{23}$ ,  $\theta_{13}$ , a CP phase  $\delta$ , and two mass squared differences  $\Delta m_{21}^2$ ,  $\Delta m_{31}^2$ . The parameters ( $|\Delta m_{31}^2|$ ,  $\theta_{23}$ ) and ( $\Delta m_{21}^2$ ,  $\theta_{12}$ ) were determined by the experiments of atmospheric and accelerator neutrinos, and by those of solar and long baseline reactor neutrinos, respectively [1]. On the other hand, the value of  $\theta_{13}$  was determined recently by the two accelerator neutrino experiments, T2K [2] and MINOS [3] and by the three reactor neutrino experiments, Double-CHOOZ [4], Daya Bay [5] and Reno [6]. The results of the global analysis are given in Refs. [7, 8, 9].

The quantities which are not yet determined are the pattern of mass hierarchy (in other words the sign of  $\Delta m_{31}^2$ ), the CP phase  $\delta$  and the octant of  $\theta_{23}$  (in other words the sign of  $\theta_{23} - \pi/4$ ).<sup>1</sup> The next things to do is to determine the pattern of mass hierarchy and the octant of  $\theta_{23}$ , before we achieve the final goal of the neutrino oscillation study, i.e., measurement of the CP phase  $\delta$ . These tasks are expected to be done in the future experiments. These experiments include so-called super beam experiments (such as nova [11], T2K

phase 2 [12, 13, 14], LBNE [15], LBNO [16]), in which neutrinos are produced in pion decays, a neutrino factory [17, 18], in which neutrinos are produced in muon decays, or a beta beam [19], in which neutrinos are produced in beta decays of radioactive isotopes.

## 2. Parameter degeneracy of the neutrino oscillation parameters

In the accelerator neutrino experiments, the appearance oscillation probabilities  $P(\nu_\mu \rightarrow \nu_e)$  and  $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$  ( $P(\nu_e \rightarrow \nu_\mu)$  and  $P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$  in the case of a neutrino factory and a beta beam) are measured, and one would naively expect that the oscillation probabilities of these two channels are sufficient to determine the CP phase  $\delta$ . It is known, unfortunately, that even if the values of the oscillation probabilities  $P(\nu_\mu \rightarrow \nu_e)$  and  $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$  are exactly given, we cannot determine uniquely the values of the oscillation parameters due to eightfold parameter degeneracy. This eightfold parameter degeneracy [20] consists of three kinds of degeneracies, the intrinsic degeneracy [21], the sign degeneracy [22] ( $\Delta m_{31}^2 \leftrightarrow -\Delta m_{31}^2$ ) and the octant degeneracy [23] ( $\theta_{23} \leftrightarrow \pi/2 - \theta_{23}$ ).

To see the eightfold degeneracy, it is convenient for the plot to give eight different points for different eight solutions. In Ref. [24] it was shown that the solution specified by  $P \equiv P(\nu_\mu \rightarrow \nu_e) = \text{const.}$  and  $\bar{P} \equiv$

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<sup>1</sup>It was reported in Refs. [10, 7, 8, 9] that the atmospheric neutrino data suggests a slight hint of deviation of  $\theta_{23}$  from  $\pi/4$ . However, we need more statistics before we draw any conclusion.

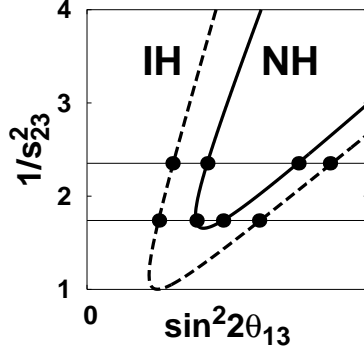


Figure 1: The eight-fold degeneracy in the  $(\sin^2 2\theta_{13}, 1/s_{23}^2)$  plane. When the two probabilities  $P(\nu_\mu \rightarrow \nu_e) = \text{const.}$  and  $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = \text{const.}$  are given, it gives us a quadratic curve for each mass hierarchy. NH (solid) and IH (dashed) stand for normal ( $\Delta m_{31}^2 > 0$ ) and inverted ( $\Delta m_{31}^2 < 0$ ) hierarchy, respectively.

$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = \text{const.}$  gives a quadratic curve (a hyperbola in most cases) in the  $(\sin^2 2\theta_{13}, 1/s_{23}^2)$  plane. In Fig.1,  $P = \text{const.}$  and  $\bar{P} = \text{const.}$  gives two different quadratic curves (due to the sign degeneracy) depending on the pattern of mass hierarchy, and each curve in general has two intersections (due to the intrinsic degeneracy) with one of two horizontal lines  $1/s_{23}^2 = 2/(1 \pm \sqrt{1 - \sin^2 2\theta_{23}})$  (two lines appear due to the octant degeneracy).

Each quadratic curve in Fig.1 shrinks to a straight line when the experiment is performed at the oscillation maximum  $|\Delta m_{31}^2|L/4E = \pi/2$ , and this is the case at the T2K experiment. However, the present result [2] is for the neutrino mode  $\nu_\mu \rightarrow \nu_e$  only, and  $P = \text{const.}$  gives the inside of the region bounded by some quadratic curve (the shaded area bounded by the thin quadratic curve in Fig.2), instead of a straight line. Since the T2K result comes with the experimental error, the allowed region at 90 %CL becomes (bounded by the thick quadratic curve and the line  $1/s_{23}^2 = 1$ ) much wider than that by the best-fit value in Fig.2.

In the future the T2K experiment will measure both  $P$  and  $\bar{P}$  with intense neutrino beams, and the error is expected to be reduced. Even with the reduced error, however, it is not obvious whether parameter degeneracy can be resolved. It turns out that the sign degeneracy is most serious to determine  $\delta$ . This can be seen from Fig.3, where the value of  $\sin \delta'$ , which is obtained with a wrong ansatz with respect to the intrinsic, sign, and octant degeneracies, respectively, is plotted as a function of the true value of  $\sin \delta$  in the case of T2K setup

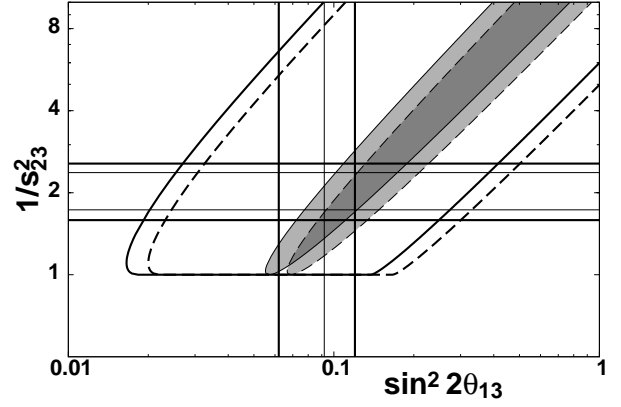


Figure 2: The allowed regions at best-fit (the shaded area bounded by the thin quadratic curve) and at 90%CL (the wider area bounded by the thick quadratic curves and the line  $1/s_{23}^2 = 1$ ) from the T2K appearance result [2]. The regions bounded by solid (dashed) curves are for normal (inverted) hierarchy. The horizontal (vertical) band  $1.6 \lesssim 1/s_{23}^2 \lesssim 2.6$  ( $0.06 \lesssim \sin^2 2\theta_{13} \lesssim 0.012$ ) bounded by thick straight lines stands for the allowed region at 90%CL of the atmospheric (reactor) neutrino experiments, respectively. The horizontal (vertical) thin straight line(s)  $1/s_{23}^2 \approx 1.7$ ,  $1/s_{23}^2 \approx 2.4$  ( $\sin^2 2\theta_{13} \approx 0.09$ ) stand for the best-fit value(s) for the atmospheric (reactor) neutrino data.

for  $\sin^2 2\theta_{13} = 0.1$  and  $\sin^2 2\theta_{13} = 0.96$ . Since the sign degeneracy can be lifted with a long baseline experiment with a baseline length  $\gtrsim 1000\text{km}$  because of a large matter effect, it is expected to be resolved by future experiments with very long baselines.<sup>2</sup>

### 3. New physics and $\nu_\tau$ detection

It is expected that the accelerator long baseline experiments with intense neutrino beams will enable us not only to measure precisely the oscillation parameters of the three flavor framework but also to probe new physics by looking for deviation from the standard scenario with three massive neutrinos.

In particular, at accelerator neutrino experiments with high energy neutrinos we can study the channels  $\nu_\mu \rightarrow \nu_\tau$  and  $\nu_e \rightarrow \nu_\tau$  with OPERA-like detectors. These experiments include the MINSIS proposal [26, 27], in which the NUMI beam is used to look for  $\nu_\tau$  at short baseline, and a neutrino factory [17, 18].<sup>3</sup>

<sup>2</sup> See Table 1 of Ref. [25] for a list of the future experiments which have a potential of resolving the sign degeneracy.

<sup>3</sup> The current baseline of the neutrino factory [28] assumes 10GeV for the muon energy and 2000km for the baseline length, and detection of  $\nu_\tau$  is expected to be difficult in this scenario. However,  $\nu_\tau$  detection may be possible in a future extension to the high energy neutrino factory.

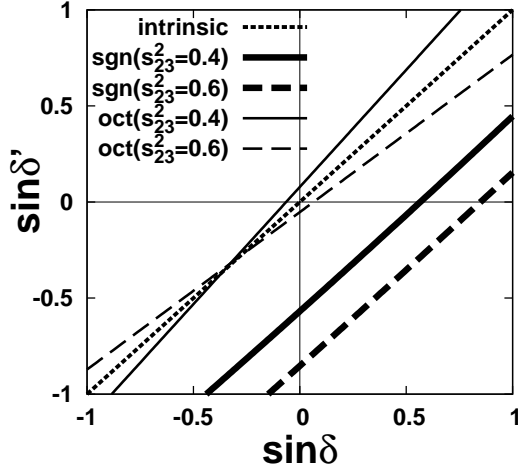


Figure 3: The dependence of the value of the CP phase  $\delta'$  which is obtained with a wrong ansatz with respect to the intrinsic, sign and octant degeneracies as a function of the true value of  $\delta$  from the T2K appearance results on  $\nu_\mu \rightarrow \nu_e$  and  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ .  $\sin^2 2\theta_{13} = 0.1$ ,  $\sin^2 2\theta_{23} = 0.96$  is assumed.

These channels are expected to give stronger constraints on new physics than those which are obtained by the present experiments. Here I will describe a few examples of new physics, where  $\nu_\tau$  detection can improve on the limits.

### 3.1. New Physics at source and detector

Let us suppose that a non-standard interaction  $\epsilon_{\alpha\beta} G_F (\bar{\nu}_\alpha \gamma_\mu \ell_\beta) (\bar{f} \gamma^\mu f')$  exists due to new physics, where  $\epsilon_{\alpha\beta}$  ( $\alpha, \beta = e, \mu, \tau$ ) is the coefficient of the non-standard interaction, normalized in terms of the weak scale,  $\nu_\alpha$  and  $\ell_\alpha$  are neutrinos and charged leptons of flavor  $\alpha$ , and  $f$  and  $f'$  stand for fermions (the only relevant ones are electrons, u and d quarks). Then this interaction predicts the exotic reactions such as  $\pi^+ \rightarrow \mu^+ + \nu_e$  or  $\nu_\mu + n \rightarrow p + \tau^-$  in the process of production and detection of neutrinos [29].

Sensitivity to  $\epsilon_{\alpha\beta}$  has been investigated by several groups. Among others, Ref. [31] showed that the sensitivity of a neutrino factory to  $\epsilon_{\alpha\tau}$  ( $\alpha = e, \mu$ ) is excellent.<sup>4</sup>

### 3.2. Violation of unitarity

It is known [33] that in generic see-saw models the kinetic term gets modified after integrating out the right handed neutrino and unitarity is expected to be violated. In the case of the so-called minimal unitarity violation,

in which only three light neutrinos are involved and sources of unitarity violation are assumed to appear only in the neutrino sector, unitarity violation is strongly constrained. Its constraint mostly comes from the bounds of rare decays of charged leptons, and deviation from unitarity is smaller than  $O(1\%)$  [33].

When the mixing matrix  $N$  is nonunitary, deviation from unitarity is expressed as  $NN^\dagger - 1$ . Ref. [33, 34, 35] studied sensitivity to  $(NN^\dagger - 1)_{\mu\tau}$  at a neutrino factory, and the bound is stronger than that of the present one from  $\tau \rightarrow \mu\gamma$ .

### 3.3. Light sterile neutrinos

The anomaly which was announced by the LSND group [36] would imply mass squared difference of  $O(1)$  eV<sup>2</sup> if it is interpreted as a phenomenon due to neutrino oscillation  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ . The standard three flavor scheme has only two independent mass squared differences, i.e.,  $\Delta m_{21}^2 \simeq 8 \times 10^{-5} \text{eV}^2$  ( $|\Delta m_{31}^2| \simeq 2.4 \times 10^{-3} \text{eV}^2$ ) for the solar (atmospheric) neutrino oscillation. To accommodate a neutrino oscillation scheme to the LSND anomaly, therefore, the extra state should be introduced. This extra state should be sterile neutrino, which is singlet with respect to the gauge group of the Standard Model, because the number of weakly interacting light neutrinos should be three from the LEP data [1]. To test the LSND anomaly, the MiniBooNE experiment has been performed, but their results [37, 38] seem to be inconclusive. On the other hand, the flux of the reactor neutrino was recalculated in Ref. [39] and it was claimed that the normalization is shifted by about +3% on average. This claim is qualitatively consistent with an independent calculation in Ref. [40]. If their claim on the reactor neutrino flux is correct, then neutrino oscillation with  $\Delta m^2 \gtrsim 1 \text{eV}^2$  may be concluded from a re-analysis of 19 reactor neutrino results at short baselines [41]. This is called reactor anomaly. Furthermore, it was pointed out in Ref. [42] that the measured and predicted <sup>71</sup>Ge production rates differ in the Gallium radioactive source experiments GALLEX and SAGE, and this is called Gallium anomaly. The anomalies of LSND, reactor and Gallium constitute the main motivation for study of sterile neutrino oscillations.

Sensitivity to sterile neutrino mixing at the MINSIS proposal was studied in Ref. [27], and it was shown that MINSIS can improve on the current bound by a factor 100 for the  $\nu_\mu \rightarrow \nu_\tau$  channel. In a neutrino scheme with one sterile neutrino state, there are three CP phases, and it was shown in Ref. [43] that the largest CP violation can potentially occur in the  $\nu_\mu \rightarrow \nu_\tau$  channel with the present constraints from other experiments. Recently a project called nuSTORM [44] was proposed as a low

<sup>4</sup> See Ref. [32] for the bounds on  $\epsilon_{\alpha\beta}$  by various experiments.

energy prototype for a neutrino factory. Although the low energy neutrino beam of nuSTORM does not allow us to create  $\tau$ 's, it has a good sensitivity to the sterile neutrino mixing angles.

#### 4. Summary

In the standard framework of three massive neutrinos, all the three mixing angles have been determined, and the remaining quantities to be measured are the sign of  $\Delta m_{31}^2$ , the sign of  $\theta_{23} - \pi/4$ , and the CP phase  $\delta$ . These quantities are expected to be determined in the future experiments, by resolving parameter degeneracies. If the neutrino energy is sufficiently high as in the MINSIS proposal or the high energy option of a neutrino factory, then one can detect  $\nu_\tau$  with an OPERA-like detector. In this case, we can probe new physics, such as non-standard interactions, unitarity violation, or light sterile neutrino scenarios, so  $\nu_\tau$  detection in neutrino oscillations deserves further study.

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